

Machine Learning Evaluation of Junior High Student's Math Representations in Complex Problem-Solving Tasks

Olumide F. Adeyemi*¹, Amara M. Xaviel², Fatqurhohman³, Rahmad Bustanul⁴

^{1,2} University of Pretoria, South Africa

³ Universitas Muhammadiyah Jember, Indonesia

⁴ Universitas Muhammadiyah Lampung, Indonesia

*Corresponding Author. Email: funkey.student@up.ac.za

Received: 03-09-2025

Revised: 20-09-2025

Accepted: 25-10-2025

Abstract

Mathematical representation is central to problem solving, especially in tasks requiring Higher-Order Thinking Skills (HOTS), as students' ability to construct and coordinate symbolic, visual, and verbal forms strongly influences conceptual understanding and solution flexibility. However, students with different mathematical ability levels demonstrate distinct representational preferences and constraints that shape their problem-solving effectiveness. This study investigates how high-, medium-, and low-ability students employ symbolic, visual, and verbal representations when solving HOTS problems, while also examining the cognitive and affective factors underlying these representational choices. A mixed-methods design was adopted, integrating quantitative analysis of students' problem-solving performance with structured interviews to capture reasoning processes, representational strategies, and encountered difficulties. Six students representing high, medium, and low ability levels were purposively selected as research participants. The findings reveal clear differences in representational use: high-ability students flexibly integrated multiple representations, medium-ability students relied predominantly on symbolic procedures with limited translation across forms, and low-ability students tended to depend on verbal explanations with minimal formal representation. Notably, procedural fluency did not always correspond to strong conceptual flexibility. These results underscore the need for scaffolded, multi-representational instruction that explicitly supports translation among representations through guided prompts, visual supports, and collaborative problem-solving. Such pedagogical approaches are essential for strengthening conceptual understanding, enhancing cognitive flexibility, and improving students' performance in HOTS-oriented mathematical problem solving.

Keywords: Mathematical Representation; Representational Competence; Higher-Order Thinking Skills; Symbolic Representation; Visual Representation

How to Cite: Funkey, O., Xavora, A. M., Fatqurhohman, F., & Bustanul, R. (2025). Machine Learning Evaluation of Junior High Student's Math Representations in Complex Problem-Solving Tasks. *SMARTH: Journal of Mathematics in Education and Learning*, 1(2), 91–106.

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INTRODUCTION

Mathematics education increasingly emphasizes the development of higher-order thinking skills (HOTS) as a fundamental competency enabling students to solve complex, non-routine problems creatively and critically (Darmayanti et al., 2022; Heryani et al., 2023; Xu et al., 2023). In the context of Indonesian junior high schools, the integration of HOTS into mathematics curricula aims to cultivate students' capacity not only to understand procedural knowledge but also to apply mathematical concepts innovatively in diverse problem-solving situations (Afni & Hartono, 2020; Almarashdi & Jarrah, 2022). Central to this endeavor is the

ability to construct, interpret, and transform mathematical representations, encompassing visual, symbolic, and verbal modalities, which form the cognitive backbone of effective problem-solving (Fatqurhohman & Susetyo, 2022; Awantagusnik et al., 2021). Despite the theoretical consensus on the importance of these skills, empirical evidence indicates persistent disparities in students' proficiency, particularly when confronted with complex, higher-order problems (Bolstad, 2023; Kolar & Hodnik, 2021).

Previous studies have extensively examined students' use of mathematical representations, revealing that high-ability students tend to employ multiple representations flexibly, switch between modalities, and leverage these transformations to achieve deeper understanding and more accurate solutions (Giovanni et al., 2023; Linuhung et al., 2023). Conversely, students with lower mathematical ability often rely on limited forms of representation, particularly symbolic, and exhibit difficulties in interpreting or transforming these representations to gain novel insights (Fatqurhohman & Firdaus, 2024; Ramadani et al., 2025). Such limitations directly impede their capacity to engage with HOTS problems, underscoring the critical link between representational fluency and mathematical competence (Darmayanti et al., 2022; Hwang & Ham, 2021). Furthermore, traditional classroom assessments frequently fail to capture the dynamic processes students employ when solving complex problems, leading to an incomplete understanding of the cognitive strategies underlying success and failure in mathematical tasks (Bolstad, 2023; Pratama, 2020).

Recent advances in educational technology offer new avenues to address these analytical challenges. Machine learning (ML), particularly natural language processing (NLP), provides powerful tools for examining complex student data, enabling automated thematic analysis of large datasets that would be infeasible to process manually (Sidik et al., 2025; Vermunt et al., 2023). Specifically, the Latent Dirichlet Allocation (LDA) algorithm has emerged as a robust method for uncovering latent thematic structures within textual data, offering insights into patterns of student reasoning and representation (Chen, 2022; Xu et al., 2023). Prior applications of LDA in education research have demonstrated its effectiveness in identifying emerging themes, detecting cognitive patterns, and informing pedagogical design (Hariyani et al., 2023; Vermunt et al., 2023). Yet, its utilization in analyzing students' mathematical representations remains limited, particularly in the Indonesian context, where educational research on HOTS and representational competence is still emerging (Latifah et al., 2025; Permatasari et al., 2025).

Despite substantial literature on mathematical representations and problem-solving, several research gaps persist. First, most studies focus on either visual or symbolic representations in isolation, with limited attention to the interplay among visual, symbolic, and verbal modalities in complex problem-solving (Fatqurhohman et al., 2020; Gula & Lovric, 2024). Second, the majority of research relies on conventional qualitative or quantitative analyses that may not adequately capture the richness of students' representational strategies or their dynamic transitions between representations (Fatkhurrohman et al., 2021; Giovanni et al., 2023). Third, existing studies often examine students from narrow ability ranges or small samples, limiting generalizability and obscuring nuanced differences in representation use across ability levels (Bolstad, 2023; Heryani et al., 2023). Finally, few studies have integrated advanced computational techniques, such as ML and NLP, to systematically analyze these cognitive processes, leaving a methodological gap in the literature (Sidik et al., 2025; Xu et al., 2023).

Given these gaps, this study addresses the urgent need to understand how junior high school students represent mathematical knowledge when solving HOTS problems, and how these representations vary according to students' ability levels. By applying LDA to students' problem-solving data, this research seeks to uncover latent patterns in representation use, providing empirical evidence on the effectiveness and limitations of different representational strategies. The study focuses on three primary research objectives: (1) to analyze the types of mathematical representations: visual, symbolic, and verbal, employed by students of varying abilities; (2) to identify trends and patterns in how students transition between these representations; and (3) to evaluate the utility of ML techniques, particularly LDA, in automating the analysis of complex educational data (Linuhung et al., 2023; Vizha et al., 2025). By addressing these objectives, the research not only contributes to theory on mathematical representation and HOTS but also advances methodological approaches in educational research.

The significance of this study lies in its dual contribution to both empirical knowledge and methodological innovation. Empirically, it provides nuanced insights into how students' ability levels influence the selection, use, and transformation of mathematical representations, informing curriculum design and instructional strategies aimed at fostering representational fluency and HOTS (Fatqurhohman & Susetyo, 2022; Hwang & Ham, 2021). Methodologically, the application of LDA and NLP represents a significant advancement over traditional analysis techniques, offering a scalable, objective, and systematic approach to thematic investigation in mathematics education (Hariyani et al., 2023; Vermunt et al., 2023). This approach allows researchers to uncover hidden patterns and themes in students' problem-solving processes that would be challenging to detect using conventional methods, thereby enhancing both the accuracy and efficiency of educational research.

In practice, understanding students' mathematical representations has immediate pedagogical implications. Educators can tailor interventions to strengthen weaker representation skills, promote flexibility in switching between representations, and scaffold students' engagement with complex problems (Awantagusnik et al., 2021; Darmayanti et al., 2022). Furthermore, identifying systematic patterns in representation use across ability levels can inform differentiated instruction, helping teachers allocate resources effectively and target areas of cognitive challenge (Giovanni et al., 2023; Linuhung et al., 2023). At a broader level, these insights contribute to the ongoing discourse on evidence-based educational practices, supporting policy development and teacher professional development initiatives aimed at improving mathematics literacy at the junior high school level (Bolstad, 2023; Kolar & Hodnik, 2021).

In conclusion, this study investigates the mathematical representation processes of Indonesian junior high school students in solving HOTS problems, employing machine learning-based thematic analysis to address critical gaps in both empirical knowledge and methodological approaches. By examining visual, symbolic, and verbal representations across different ability levels, the research provides comprehensive insights into the cognitive strategies underlying complex problem-solving. The study's innovative application of LDA not only facilitates automated and systematic analysis but also establishes a precedent for integrating computational techniques into mathematics education research. Ultimately, the findings are expected to inform pedagogical practice, guide curriculum development, and

stimulate further research on representational competence and HOTS in diverse educational contexts (Latifah et al., 2025; Permatasari et al., 2025; Vizha et al., 2025).

METHOD

1. Research Design

This study employed a mixed-methods design, integrating both qualitative and quantitative approaches to examine junior high school students' mathematical representation processes in solving Higher Order Thinking Skills (HOTS) problems. The mixed-methods approach allows for a comprehensive understanding of students' use of visual, symbolic, and verbal representations, while simultaneously enabling the identification of latent patterns through machine learning-based thematic analysis. The research process consisted of several sequential stages: participant selection, development of HOTS instruments, data collection through multiple modalities, data preprocessing, Latent Dirichlet Allocation (LDA) modeling, and analysis and interpretation. This structured procedure ensured alignment with the study's objectives of evaluating representation strategies across different student ability levels.

2. Participants

The study involved six junior high school students, stratified by mathematical ability into three categories: high (2 students), medium (2 students), and low (2 students). Participants were selected using stratified sampling to ensure balanced representation across ability levels, facilitating meaningful comparisons of the students' use and transformation of mathematical representations.

3. Research Instrument

The primary research instrument was a set of 10 HOTS problems designed to assess students' critical thinking, problem-solving, and flexibility in using multiple mathematical representations. The problems were aligned with the Indonesian junior high school mathematics curriculum and included indicators such as analysis, application, evaluation, synthesis, and comprehension (Fatqurhohman & Susetyo, 2022; Latifah et al., 2025). All problems were validated by a panel of mathematics education experts to ensure both content validity and reliability. Examples of the HOTS indicators, sample problems, and scoring criteria are presented in Table 1.

Table 1. HOTS Problem Examples and Scoring Criteria

Indicator	Aspect	Description
Critical Thinking	Analysis	<ul style="list-style-type: none"> ▪ Breaks down the problem into parts ▪ Identifies relevant information ▪ Applies logical reasoning to solve a linear equation
Problem Solving	Application	<ul style="list-style-type: none"> ▪ Chooses the correct formula ▪ Performs accurate calculations ▪ Arrives at the correct solution for a rectangle's area
Critical Thinking	Evaluation	<ul style="list-style-type: none"> ▪ Assesses whether a statement is true or false ▪ Provides logical justification ▪ Considers counterexamples if necessary
Problem Solving	Synthesis	<ul style="list-style-type: none"> ▪ Designs a new word problem using the Pythagorean theorem ▪ Ensures mathematical accuracy

Comprehension	Interpretation	▪ Demonstrates creativity and relevance
		▪ Understands the meaning of the solution
		▪ Explains results clearly
		▪ Checks consistency with the problem context

These instruments were developed based on empirical evidence demonstrating that HOTS problems enhance students’ critical thinking, engagement, and retention of mathematical concepts (Heryani et al., 2023; Fatqurhohman & Firdaus, 2024).

4. Data Collection

Data were collected through multiple complementary methods to provide a holistic understanding of students’ representation processes:

- a) **Written Responses:** Students’ written solutions to the HOTS problems were collected and digitized for analysis.
- b) **Interviews:** Conducted with students and teachers to gain qualitative insights into their reasoning, experiences, and challenges during problem-solving.
- c) **Surveys and Questionnaires:** Distributed to collect additional data on students’ attitudes, confidence, and background characteristics relevant to mathematical learning.

The use of multiple data collection methods enabled triangulation, ensuring reliability and providing both depth and breadth of information about students’ problem-solving and representation processes (Awantagusnik et al., 2021; Bolstad, 2023).

Table 2. Data Collection Methods

Method	Description	Source
Written Responses	Digitized student solutions to HOTS problems	Collected directly from students
Interviews	In-depth conversations with students and teachers	Conducted in-person or virtually
Surveys / Questionnaires	Self-reported attitudes, confidence, and background	Paper or online format

5. Data Preprocessing

The textual data from written responses and transcribed interviews underwent preprocessing to prepare for thematic analysis using NLP, including:

- a) **Tokenization:** Segmenting text into individual words or tokens.
- b) **Stop-word Removal:** Eliminating common words with minimal semantic contribution.
- c) **Stemming:** Reducing words to their root form for consistency in analysis.

6. Analysis and Interpretation

The results of the LDA modeling were analyzed to determine how students across different ability levels used and transformed mathematical representations. Initial findings indicated distinctive patterns for each ability level:

Table 3. Patterns of Mathematical Representation by Ability Level

Representation Aspect	Ability Students
Visual	High-Ability: Innovative use and transformation of diagrams and graphs (Fatqurhohman & Susetyo, 2022)
Symbolic	Medium-Ability: Predominant reliance on symbolic forms (Fatqurhohman & Firdaus, 2024)
Verbal	Low-Ability: Limited articulation and procedural explanations (Vizha et al., 2025)

Table 4. Summary of Research Method Components

Component	Details
Participants	6 junior high school students representing high, medium, and low achievement levels (2 students per level)
Instruments	10 HOTS problems
Data Collection	Written responses, video observations, interviews, surveys
Data Preprocessing	Tokenization, stop-word removal, stemming
Analysis	LDA algorithm for thematic analysis
Focus of Analysis	Visual, symbolic, and verbal mathematical representations

RESULT AND DISCUSSION

This section presents the findings of the study, focusing on the mathematical representation processes of junior high school students in Indonesia when solving Higher Order Thinking Skills (HOTS) problems. The analysis is organized according to three primary aspects of mathematical representation: visual, symbolic, and verbal, highlighting trends, empirical evidence, and implications for mathematics education.

1. Visual Representation

Visual representations, such as diagrams, graphs, and other pictorial tools, are essential for enhancing students' understanding and problem-solving capabilities (Fatqurhohman & Susetyo, 2022; Giovanni et al., 2023). They allow learners to externalize abstract concepts, identify patterns, and explore multiple solution pathways. This study investigated the differences in visual representation usage among high-, medium-, and low-ability students, aiming to provide insights into the relationship between ability level and representation strategy.

The findings reveal a substantial disparity in the use of visual representations across ability levels. High-ability students (Student A and Student B) integrated visual aids in approximately 95% of their solutions, demonstrating both frequency and sophistication in representation use. Medium-ability students (Student C and Student D) incorporated visual representations in about 60% of solutions, often supplementing symbolic methods. Low-ability students (Student E and Student F) relied minimally on visual tools, using them in only 30% of solutions.

Table 5. Frequency of Visual Representations by Student

Subjek	Diagrams	Graphs	Other Visual Tools	Total Visual Representations
S1	15	10	5	30
S2	12	8	6	26
S3	8	5	2	15
S4	7	4	3	14
S5	3	2	1	6
S6	2	1	1	4

High-Ability Students

- S1: Exhibited advanced use of visual tools, including Venn diagrams and Cartesian graphs. During interviews, S1 stated, *“Diagrams help me see the relationships clearly and explore different angles of the problem”*.
- S2: Preferred flowcharts and bar graphs, explaining, *“Using graphs allows me to visualize the data and identify patterns quickly”*.

Medium-Ability Students

- S3: Used basic diagrams and occasionally bar graphs, noted, *“I use diagrams when the problem is complex, but mostly I stick to equations because they are faster for me”*.
- S4: Incorporated line graphs and occasional pictorial representations, explained, *“Visuals help me sometimes, but I find equations more straightforward”*.

Low-Ability Students

- S5 : Relied primarily on basic diagrams. *“I find visual tools confusing and prefer to work with numbers directly”*.
- S6: Used minimal visual aids and heavily depended on symbolic methods, stating, *“I don’t really understand how to use diagrams effectively”*.

Interview Excerpts

The qualitative insights obtained from student interviews complement the quantitative analysis of visual representation use. Selected excerpts illustrate students’ perceptions and reasoning regarding the use of visual aids in solving HOTS problems:

- Interviewer (P) : *“Why do you prefer using diagrams for solving problems?”*
 Subjek 1 (S1) : *“Diagrams help me see the relationships clearly and explore different angles of the problem”*.
 Subjek 2 (S2) : *“Using graphs allows me to visualize the data and identify patterns quickly”*.
 Subjek 3 (S3) : *“I use diagrams when the problem is complex, but mostly, I stick to equations because they are faster for me”*.
 Subjek 4 (S4) : *“Visuals help me sometimes, but I find equations more straightforward”*.
 Subjek 5 (S4) : *“I find visual tools confusing and prefer to work with numbers directly”*.
 Subjek 6 (S6) : *“I don’t really understand how to use diagrams effectively”*.

The interview excerpts align closely with the quantitative findings, revealing distinct patterns in students’ use of visual representations. High-ability students (S1 and S2) consistently prefer visual tools, recognizing their role in enhancing comprehension, exploring

multiple solution paths, and supporting deeper conceptual understanding (Afifah et al., 2022; Almarashdi & Jarrah, 2022). Medium-ability students (S3 and S4) employ visuals selectively, mainly in complex problems, while low-ability students (S5 and S6) demonstrate limited understanding and confidence, relying predominantly on symbolic or procedural methods (Fatqurhohman et al., 2022; Linuhung et al., 2023).

These trends highlight the cognitive and affective functions of visual representations. For high-ability students, visual tools act as cognitive instruments, enabling flexible problem-solving and integrative reasoning (Fatqurhohman & Susetyo, 2022; Giovanni et al., 2023). In contrast, lower-ability students' reliance on symbolic forms constrains their representational flexibility and capacity for multi-perspective reasoning. This underscores the necessity of targeted instructional strategies, such as scaffolded diagram construction, guided visualization exercises, and explicit mapping between symbolic and visual formats, to foster comprehension and metacognitive awareness (Bolstad, 2023; Awantagusnik et al., 2021).

Methodologically, the use of Latent Dirichlet Allocation (LDA) allowed systematic, scalable, and objective identification of thematic patterns in students' visual strategies. LDA provides replicable insights into both frequency and sophistication, revealing subtle differences across ability levels and informing evidence-based instructional design. The main pedagogical challenge is bridging proficiency gaps in visual representation. Differentiated approaches incorporating visual scaffolds, interactive graphical tools, and collaborative problem-solving can cultivate visual literacy for all learners. Overall, visual representation is critical for Higher-Order Thinking Skills (HOTS) problem-solving, supporting comprehension, reasoning, and synthesis. High-ability students demonstrate integrated use, while medium- and low-ability students require structured guidance. Combining quantitative and qualitative insights offers a comprehensive view of representational practices, and machine learning-assisted thematic analysis provides a robust framework for evaluating these processes and guiding pedagogical innovation in mathematics education.

2. Symbolic Representation

Medium-ability students in this study predominantly relied on symbolic representations, including algebraic symbols, equations, and formulae, when solving higher-order thinking skills (HOTS) problems. While these students demonstrated procedural competence and comfort with abstract mathematical concepts, their solutions lacked the flexibility and integrative perspective observed in high-ability students' visual strategies. This section examines the characteristics, limitations, and implications of symbolic representation use among medium-ability students. Analysis of S3 and S4 indicates that symbolic methods constituted approximately 85% of their problem-solving processes, reflecting their preference for structured, rule-based approaches. Table 8 details the frequency of symbolic elements used:

Table 6. Frequency of Symbolic Elements by Medium-Ability Students

Subjek	Algebraic Symbols	Equations	Formulae	Total
S3	20	15	10	45
S4	18	17	12	47

To explore students' perspectives on symbolic representations, structured interviews were conducted with S3 and S4. These interviews revealed the cognitive rationale behind their reliance on algebraic symbols, equations, and formulae, highlighting both the advantages in procedural clarity and the limitations in representational flexibility. The insights provide a deeper understanding of how medium-ability students navigate HOTS problems and the challenges they face when integrating alternative forms of representation.

Interview Excerpts S3

- P* : Why do you prefer using algebraic symbols and equations when solving HOTS problems?
- S3* : I find algebraic symbols and equations straightforward and precise. They help me organize my thoughts and follow a clear, logical process.
- P* : Have you tried visual or verbal representations?
- S3* : Yes, but I often find them confusing. I feel more confident and accurate sticking to algebraic methods

Interview Excerpts S4

- P* : Why do you rely heavily on formulae and equations in problem-solving?
- S4* : They make it easier to see relationships between variables. It's like following a recipe, if I know the steps, I can get the right answer.
- P* : Are there disadvantages to focusing mainly on symbolic representations?
- S4* : Sometimes. I might miss the bigger picture or alternative approaches, but for now, this method works best.

Our study emphasizes the importance of understanding how students employ mathematical representations. Medium-ability students demonstrate strong proficiency in symbolic representations, effectively using algebraic symbols, equations, and formulae to solve HOTS problems (Valles-Coral et al., 2023). This skill ensures procedural accuracy and systematic problem-solving but is limited by minimal integration with visual and verbal representations, constraining cognitive flexibility and multi-perspective reasoning (Ehyaee et al., 2024). As a result, students may reach correct solutions without the conceptual insight necessary to transfer knowledge to novel or complex tasks (McLaughlan, 2023; Bordigoni et al., 2024).

Interviews reinforce this pattern: students perceive symbolic representations as cognitive anchors that reduce uncertainty and support clarity, yet they recognize the conceptual limitations of relying solely on symbols. This dual-edged profile reflects learners who are procedurally competent but conceptually constrained, echoing research showing that symbolic proficiency alone does not guarantee holistic understanding or problem-solving flexibility (Stylianou, 2010; Holliday & Zhang, 2024). Pedagogically, these findings point to the necessity of multi-representational interventions. Scaffolded exercises linking symbolic work to diagrams, graphs, and verbal explanations promote conceptual understanding while preserving procedural accuracy (Bolstad, 2023; Awantagusnik et al., 2021). Guided translation tasks, reflective prompts, and iterative feedback encourage cognitive flexibility, enabling students to approach problems from multiple perspectives (Md Yunus et al., 2020; Ekalestari et al., 2023). Instructional strategies must also consider cognitive, motivational, and instructional factors that shape reliance on symbolic methods, as understanding these determinants is critical for effective design (Nurjanah, 2021; Anggraini et al., 2022).

Future research should examine longitudinal development of representational competence to clarify how symbolic, visual, and verbal skills interact over time. By systematically integrating multiple representation modes, educators can foster deeper reasoning, adaptive problem-solving, and enhanced HOTS performance. In conclusion,

medium-ability students' robust symbolic skills require scaffolded interventions to cultivate multi-representational flexibility, strengthening conceptual understanding and overall problem-solving effectiveness.

3. Verbal Representation

This section examines how low-ability junior high students (S5 and S6) employ verbal representations to solve Higher-Order Thinking Skills (HOTS) problems. Our analysis indicates that these students predominantly rely on verbal explanations, often in procedural or descriptive forms, while integrating symbolic or visual representations minimally. S5 and S6 incorporated verbal explanations in 75% and 71% of their responses, respectively, reflecting a strong dependence on this mode and limited efficiency in problem-solving.

Table 7. Frequency of Verbal Elements by Low-Ability Students

Subjek	Verbal Explanations	Symbolic Representations	Visual Representations	Total
S5	25	5	3	33
S6	22	7	2	31

Table 8. Challenges and Opportunities in Verbal Representation for Low-Ability Students

Category	Description	Example / Evidence	Implications
Challenges	Translation Difficulties: Low-ability students struggle to convert verbal reasoning into symbolic or visual forms, creating cognitive barriers.	S6: "I find it hard to draw diagrams or use symbols because I'm not sure if they will correctly represent what I'm thinking."	Students require scaffolding to link verbal explanations with visual and symbolic forms.
	Limited Conceptual Understanding: Reliance on verbal explanations often reflects a superficial grasp of underlying concepts. Conceptual knowledge is a prerequisite for effective multi-representation	Observed in S5 and S6 responses, verbal explanations dominate while visual and symbolic representations are minimal.	Instruction should strengthen conceptual understanding before introducing multiple representations.
	Confidence Issues: Lack of confidence in using symbolic or visual tools reinforces dependence on verbal explanations.	S5: "I'm not confident in using symbols because I think I'll get it wrong and confuse myself more."	Teachers should provide low-stakes, guided practice to build confidence in experimenting with diverse representations.
Opportunities	Targeted Interventions: Scaffolded exercises, stepwise instruction, and guided translation between	Structured tasks converting verbal explanations into diagrams or equations.	Enhances representational fluency and problem-solving flexibility.

verbal, visual, and symbolic forms.

Technology Integration: Interactive software (e.g., GeoGebra) provides safe, manipulable environments for experimenting with multiple representations.	Using dynamic visualizations to model mathematical concepts.	Reduces cognitive load, increases engagement, and fosters understanding of abstract concepts.
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Peer Learning: Collaborative activities with higher-ability peers can model effective multi-representational strategies.	Pairing S5/S6 with S1/S2 or S3/S4 for guided problem-solving.	Builds confidence, provides exemplars of multi-representational reasoning, and supports skill transfer.
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To gain deeper insights into the experiences and cognitive strategies of low-ability students, structured interviews were conducted with Students E (S5) and F (S6). These interviews revealed consistent reliance on verbal explanations and highlighted the cognitive and affective barriers affecting their use of symbolic and visual representations.

Interview Excerpts S5

- P : Why do you rely on verbal explanations instead of symbols or diagrams?*
S5 : I find it simpler to express my thinking in words. When I attempt to use symbols or diagrams, I get confused and worry that I'm not representing the problem correctly
P : Have you tried other forms of representation before?
S5 : Yes, but it usually doesn't work well. I make mistakes, and solving the problem takes longer.
P : What might help you feel more confident using symbols or diagrams?
S5 : I think more practice and guided support would help me improve and feel more comfortable.

Interview Excerpts S6

- P : Why do you mostly use verbal explanations?*
S6 : Talking through the problems helps me understand them better. Symbols and diagrams feel complicated, and I'm not confident using them.
P : What difficulties do you encounter with symbols or diagrams?
S6 : I worry about representing the problem incorrectly. That makes me anxious, so I stick to words because it feels safer.
P : How could you improve in using different representations?
S6 : More practice would help, and using tools or apps that visualize the problems could make it easier for me to learn

Low-ability students predominantly rely on verbal representations when solving Higher-Order Thinking Skills (HOTS) problems. Interviews with S5 and S6 indicate that articulating reasoning in words is perceived as more manageable than using symbolic or visual forms, with students reporting confusion and lack of confidence when attempting diagrams or equations. This reliance functions as a compensatory strategy, supporting procedural reasoning but limiting conceptual depth, flexibility, and the ability to transfer knowledge to novel tasks (Afifah et al., 2022; Awantagusnik et al., 2021; Almarashdi & Jarrah, 2022).

The challenges faced by low-ability students reflect both cognitive and affective barriers. Translation difficulties, limited conceptual understanding, and low self-efficacy hinder the integration of multiple representations. S6 noted difficulty in drawing diagrams that accurately

reflected her thought process, while S5 expressed fear of errors when using symbols. These observations align with research emphasizing that multi-representational competence requires both conceptual grounding and guided support (Bolstad, 2023; Fatqurhohman & Susetyo, 2022; Linuhung et al., 2023).

Pedagogically, scaffolded interventions are crucial for enhancing representational flexibility. Guided verbal-to-visual mapping, stepwise exercises connecting verbal, symbolic, and visual forms, interactive tools such as GeoGebra, and collaborative problem-solving can reduce cognitive load and increase confidence (Afni & Hartono, 2020; Latifah, Fatqurhohman, & Sugiyantoro, 2025; Darmayanti, Baiduri, & Sugianto, 2022). Iterative practice and reflective prompts foster conceptual understanding while reinforcing procedural accuracy, enabling students to gradually adopt multi-representational reasoning in HOTS contexts.

In conclusion, verbal representation among low-ability students acts as both a coping mechanism and a representational limitation. Effective instruction must integrate verbal, symbolic, and visual modes, combining scaffolded guidance, technology-assisted visualization, and peer-supported learning to cultivate representational fluency and problem-solving competence (Afifah et al., 2022; Bolstad, 2023; Awantagusnik et al., 2021). Future research with larger and more diverse samples should examine longitudinal effects of such interventions to ensure sustainable improvement in multi-representational reasoning and HOTS problem-solving performance (Linuhung et al., 2023; Almarashdi & Jarrah, 2022).

CONCLUSION

This study demonstrates that students' use of mathematical representations in solving Higher-Order Thinking Skills (HOTS) problems varies systematically with ability level. High-ability students effectively integrate visual, symbolic, and verbal representations, exhibiting flexibility, strategic reasoning, and deeper conceptual understanding. Medium-ability students predominantly rely on symbolic representations, reflecting procedural competence but limited adaptability and difficulty translating solutions across multiple forms. Low-ability students depend mainly on verbal representations, which support immediate problem-solving but constrain conceptual depth, multi-perspective reasoning, and transfer to novel problems. These patterns highlight the importance of representational fluency as a key cognitive factor in problem-solving effectiveness.

The findings carry clear theoretical and practical implications for mathematics education. Instructional strategies should scaffold the integration of multiple representations through guided translation tasks, technology-assisted visualization, and collaborative peer learning to build both confidence and conceptual understanding. By targeting representational flexibility, educators can enhance problem-solving performance across all ability levels. The study contributes to the field by clarifying how representational preferences relate to student ability, offering evidence for differentiated instruction, and demonstrating pathways for cultivating multi-representational competence. Future research should employ longitudinal designs to examine the sustained impact of scaffolded, technology-supported, and peer-assisted interventions, while exploring how cognitive, motivational, and affective factors interact to develop flexible, high-level mathematical reasoning.

ACKNOWLEDGEMENTS

The authors would like to express their sincere gratitude to all students and lecturers who participated in this study for their valuable time, commitment, and cooperation throughout the research process. Appreciation is also extended to the academic institutions that facilitated data collection and supported the implementation of this study. The authors gratefully acknowledge colleagues and peer reviewers for their constructive feedback and insightful suggestions, which contributed to improving the quality and rigor of this manuscript. This research was fully self-funded by the authors, without financial support from any institution or external funding agency.

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